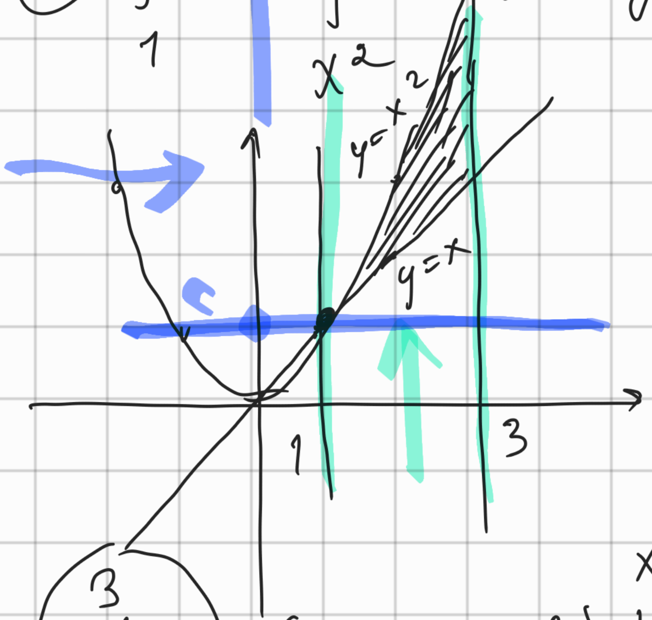
ЗК-19

5.11.20.

① $\int\limits_1^3 dx \int\limits_{x^2}^{x} (x-y)\,dy$



$1 \le x \le 3$ числа

$x^2 \le y \le x$ функции.

y - числах

x - в функциях

$$\int\limits_1^3 dx \left(xy - \frac{y^2}{2}\right)\Bigg|_{x^2}^{x} = \int\limits_1^3 \left(x^2 - \frac{x^2}{2} - x^3 + \frac{x^4}{2}\right)dx =$$

$$= \int\limits_1^3 \left(\frac{x^2}{2} - x^3 + \frac{x^4}{2}\right)dx = \frac{x^3}{6} - \frac{x^4}{4} + \frac{x^5}{10}\Bigg|_1^3 =$$

$$\frac{27}{6} - \frac{81}{4} + \frac{243}{10} - \frac{1}{6} + \frac{1}{4} - \frac{1}{10} = \frac{26}{6} - \frac{80}{4} + \frac{242}{10} =$$

$$= \frac{13}{3} - 20 + 24{,}2 = \frac{13}{3} + 4{,}2 = \frac{13}{3} + 4\frac{1}{5} =$$

$$= \frac{13}{3} + \frac{21}{5} = \frac{65+63}{15} = \frac{128}{15}$$

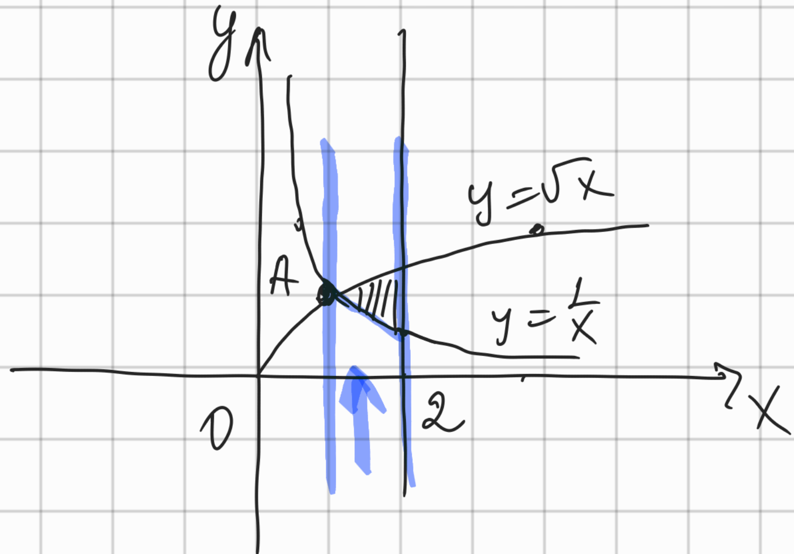
$y = x^2 \;\downarrow\; x = \sqrt{y}$ $\qquad$ $y = x \;\downarrow\; x = y$

$x = 1$ т.к. $y = x$ $\quad y = 1$

$x = 3$ т.к. $y = x^2$ $\quad y = 9$

$$\int_1^9 dy \int_{\sqrt{y}}^{y} (x-y)\,dx$$

② $\iint y \ln x \, dx\, dy$ $\qquad D: \begin{cases} xy=1 \\ y=\sqrt{x} \\ x=2 \end{cases} \to y=\frac{1}{x}$



$A:\ \begin{matrix} y=\sqrt{x} \\ y=\frac{1}{x} \end{matrix}$

$\sqrt{x}=\frac{1}{x}$

$x=\frac{1}{x^2} \qquad x^3=1 \qquad x=1$

$$D: \begin{cases} 1 \le x \le 2 \\ \frac{1}{x} \le y \le \sqrt{x} \end{cases}$$

число

$$\int_1^2 dx \int_{\frac{1}{x}}^{\sqrt{x}} y \ln x \, dy = \int_1^2 \ln x \, dx \int_{\frac{1}{x}}^{\sqrt{x}} y\, dy =$$

$$= \int_1^2 \ln x \left(\frac{y^2}{2} \Big|_{\frac{1}{x}}^{\sqrt{x}} \right) dx = \int_1^2 \underbrace{\ln x}_{u} \underbrace{\left(\frac{x}{2} - \frac{1}{2x^2} \right) dx}_{dv}$$

$$\begin{cases} u=\ln x & du=\frac{1}{x}\,dx \\ dv=\left(\frac{x}{2}-\frac{1}{2x^2}\right)dx & v=\int\left(\frac{x}{2}-\frac{1}{2x^2}\right)dx = \end{cases}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left(-\frac{1}{x}\right) = \frac{x^2}{4} + \frac{1}{2x} \Big\} =$$

$$= \left[\int u\,dv = uv - \int v\,du \right] = \left(\frac{x^2}{4} + \frac{1}{2x}\right)\ln x -$$

$$- \int \left(\frac{x^2}{4} + \frac{1}{2x}\right) \cdot \frac{1}{x}\,dx = \left(\frac{x^2}{4} + \frac{1}{2x}\right)\ln x -$$

$$- \int \left(\frac{x}{4} + \frac{1}{2x^2}\right)dx = \left(\frac{x^2}{4} + \frac{1}{2x}\right)\ln x - \frac{x^2}{8} + \frac{1}{2x}\Bigg|_1^2 =$$

$$= \left(1 + \frac{1}{4}\right)\ln 2 - \frac{1}{2} + \frac{1}{4} - \underbrace{\left(\frac{1}{4} + \frac{1}{2}\right)\ln 1}_{=0} + \frac{1}{8} - \frac{1}{2} =$$

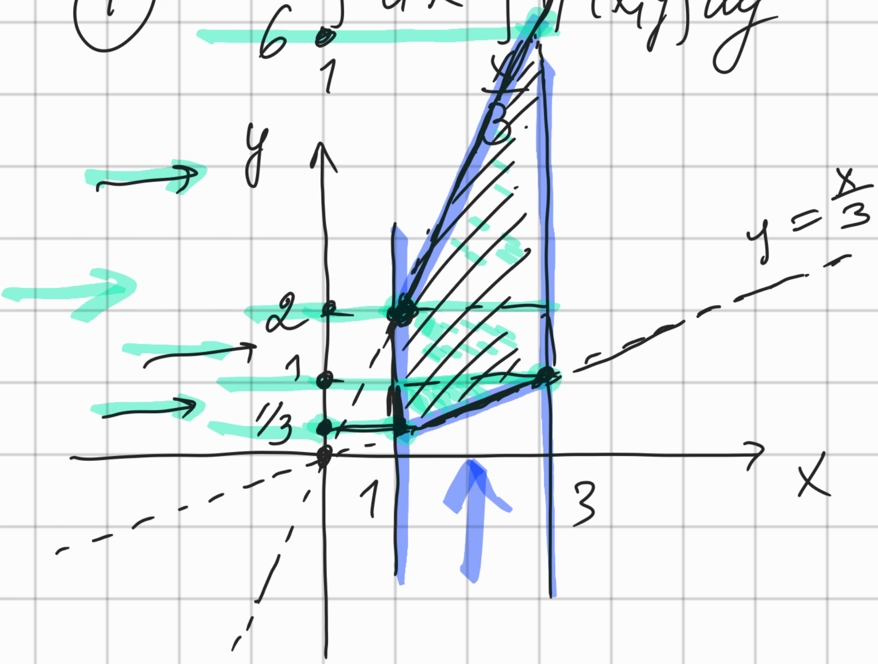
$$= \frac{5}{4}\ln 2 - 1 + \frac{3}{8} = \frac{5}{4}\ln 2 - \frac{5}{8}$$

Нарисовать область интегрирования и изменить порядок интегрирования

(1) $\int_1^3 dx \int_{x/3}^{2x} f(x,y)\,dy$

$1 \le x \le 3$

$\frac{x}{3} \le y \le 2x$



$y = \frac{x}{3}$

x	3	0
y	1	0

$y = 2x$

x	0	1
y	0	2

$y = \frac{x}{3} \to x = 3y$

$y = 2x \to x = \frac{y}{2}$

$$\int_{1/3}^{1} dy \int_{1}^{3y} f(x,y)\,dx + \int_{1}^{2} dy \int_{1}^{3} f(x,y)\,dx +$$

$$+ \int_{2}^{6} dy \int_{\frac{y}{2}}^{3} f(x,y)\,dx$$

(2) $$\int_{0}^{3} dx \int_{0}^{\sqrt{25-x^2}} f(x,y)\,dy$$

$0 \le x \le 3$

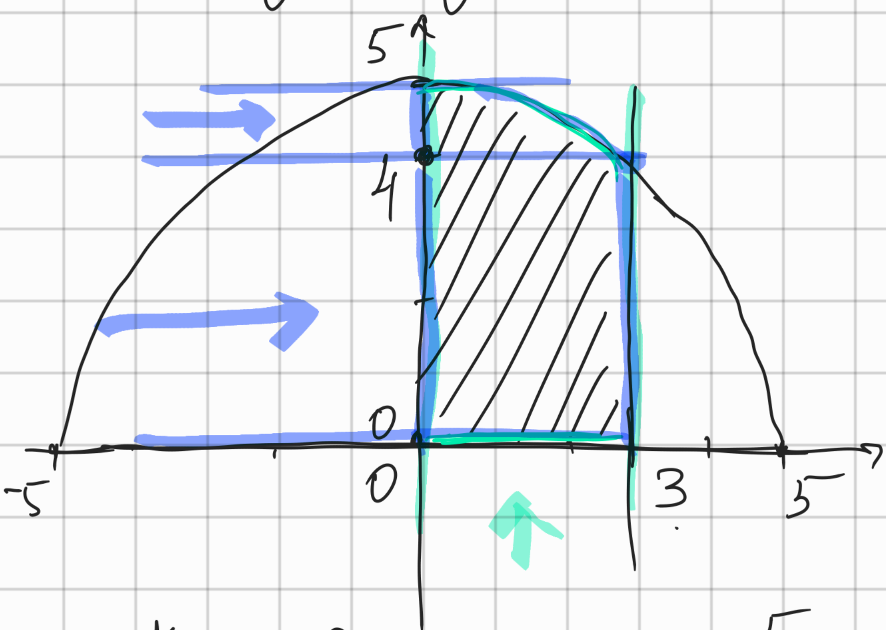
$0 \le y \le \sqrt{25-x^2}$

$y = \sqrt{25-x^2} > 0$

$y^2 = 25 - x^2$

$x^2 + y^2 = 25$ окр.

$x = \sqrt{25-y^2}$



$\left[\begin{array}{l} \text{ось } 0x \Rightarrow y = 0 \\ \text{ось } 0y \Rightarrow x = 0 \end{array}\right]$

$$\int_{0}^{4} dy \int_{0}^{3} f(x,y)\,dx + \int_{4}^{5} dy \int_{0}^{\sqrt{25-y^2}} f(x,y)\,dx$$

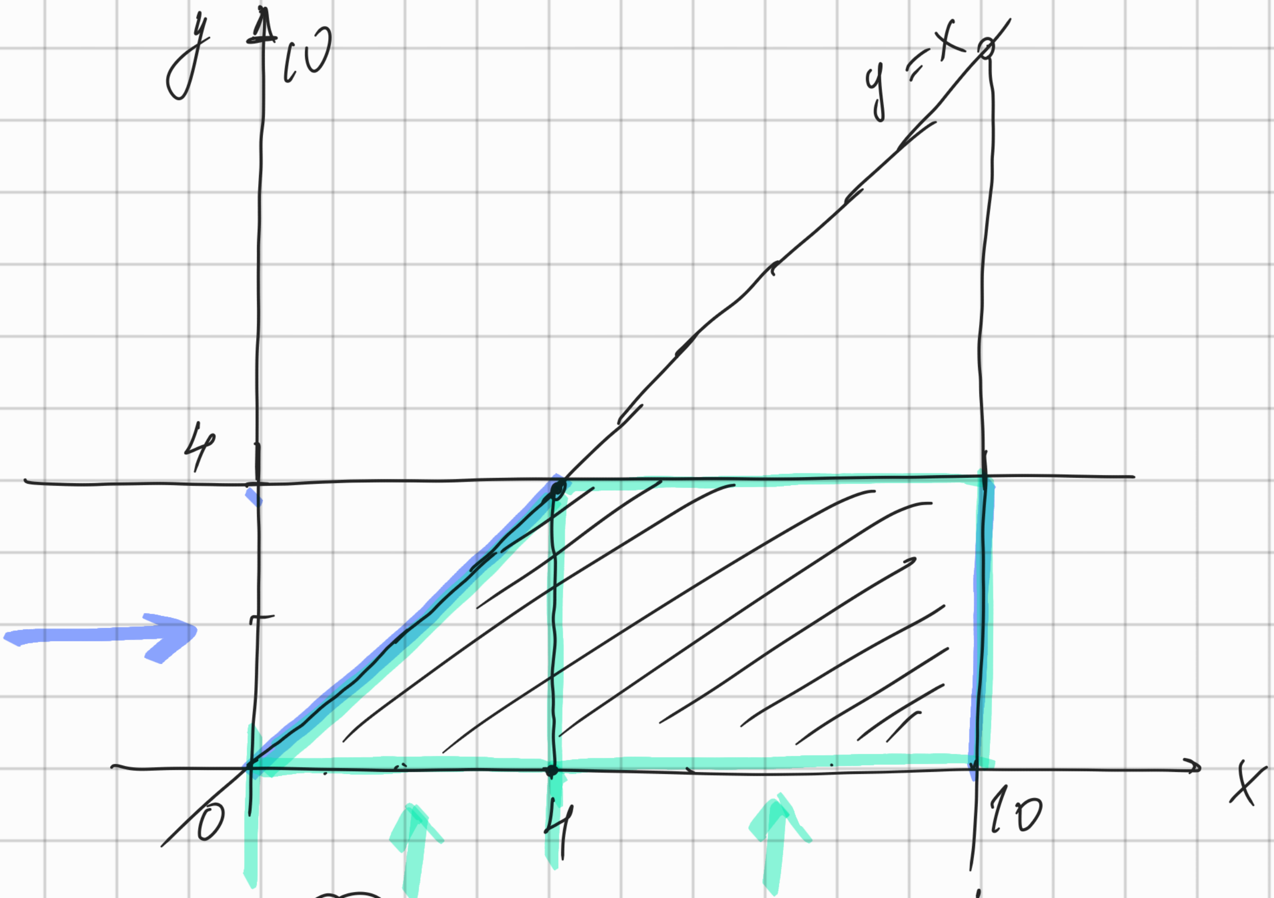
(3) $$\int_{0}^{4} dy \int_{y}^{10} f(x,y)\,dx$$

$0 \le y \le 4$

$y \le x \le 10$

$y = x$

$x = y$



$$\int_0^4 dx \int_0^{x} f(x,y)\,dy + \int_4^{10} dx \int_0^{4} f(x,y)\,dy$$

4) $$\int_0^1 dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)\,dx \qquad \left(\begin{gathered} 0 \le y \le 1 \\ \underbrace{-\sqrt{1-y^2}} \le x \le \underbrace{1-y} \end{gathered} \right)$$

$\le x \le$

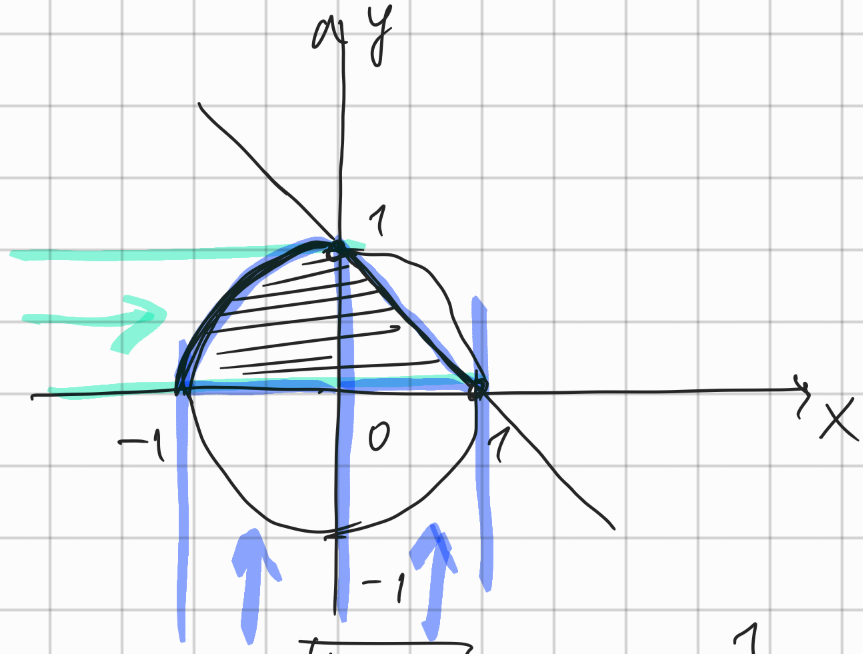
$\le y \le$

$x = 1-y \to \underline{y = 1-x}$

прямая

$\underbrace{x = -\sqrt{1-y^2}}_{x<0.}$ $\quad x^2 = 1-y^2 \quad x^2+y^2=1$ — окружность

$y = \pm\sqrt{1-x^2}$



$y = 1 - x$

x	0	1
y	1	0

$$\int_{-1}^{0} dx \int_{0}^{\sqrt{1-x^2}} f(x,y)\,dy + \int_{0}^{1} dx \int_{0}^{1-x} f(x,y)\,dy$$

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$$\int_{-1}^{1} dx \int_{x^2-1}^{\sqrt{1-x^2}} f(x,y)\,dy$$

$-1 \leq x \leq 1$

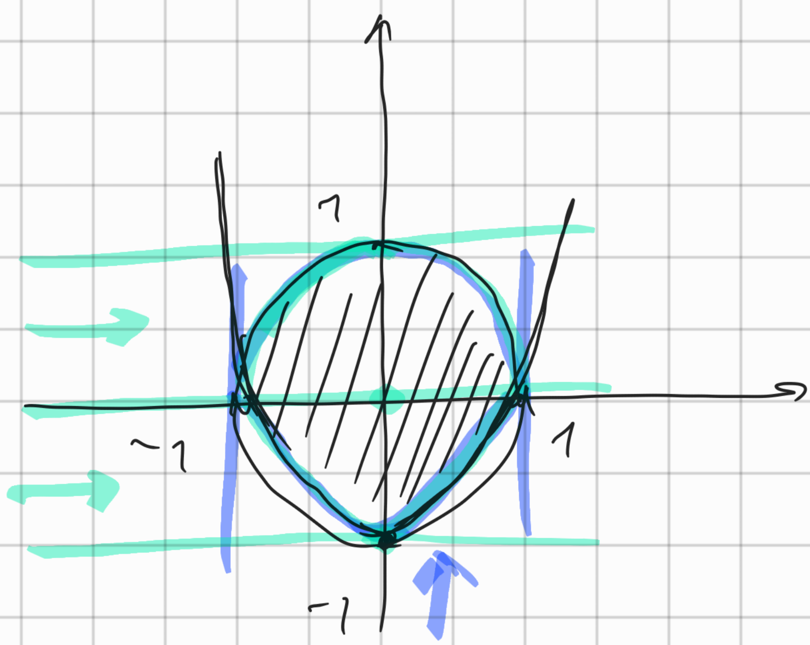
$x^2-1 \leq y \leq \sqrt{1-x^2}$

$y = x^2 - 1$ – парабола

$x = \pm\sqrt{y+1}$

$y = \sqrt{1-x^2}$ $\quad x^2+y^2=1$ окр-ть

$x = \pm\sqrt{1-y^2}$



$$\int_{-1}^{0} dy \int_{-\sqrt{y+1}}^{\sqrt{y+1}} f(x,y)\,dx + \int_{0}^{1} dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx$$

(6) $$\int_{-2}^{0} dy \int_{y^2-4}^{0} f(x,y)\,dx$$

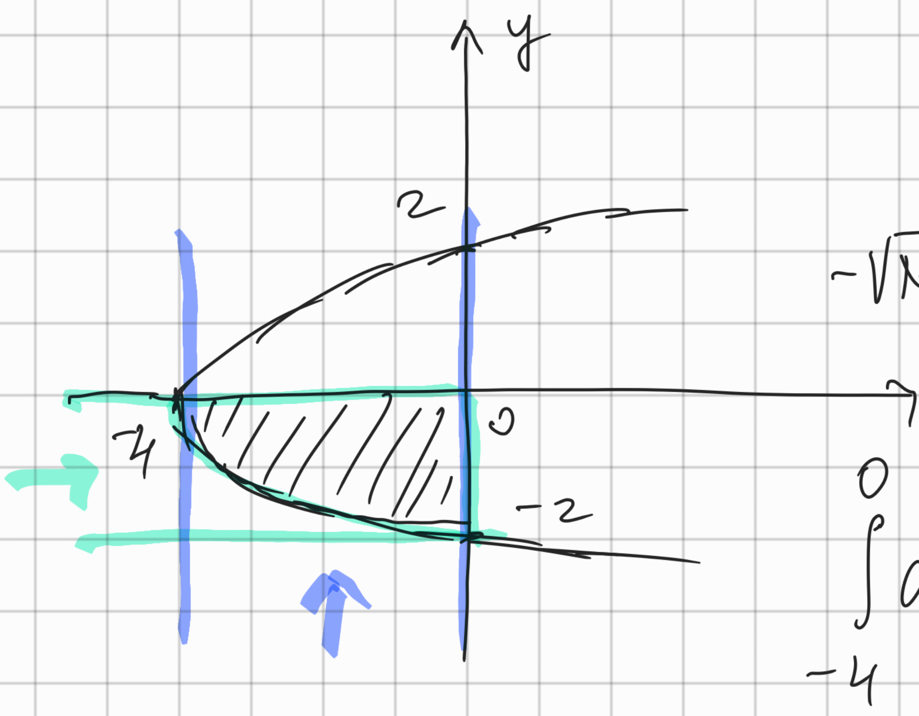
(7) $$\int_{-1}^{2} dx \int_{x^2}^{2+x} f(x,y)\,dy$$

(6) $-2 \le y \le 0$

$y^2-4 \le x \le 0$

$y^2-4=x$ – парабола ветви вправо.

$y = \pm\sqrt{x+4}$



$-4 \le x \le 0$

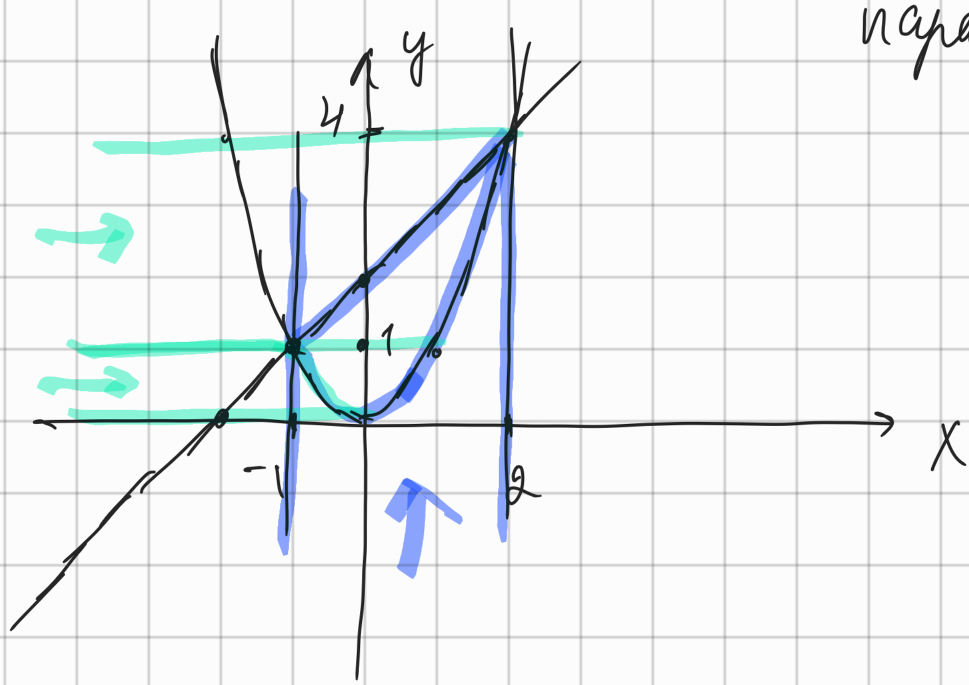
$-\sqrt{x+4} \le y \le 0$

$$\int_{-4}^{0} dx \int_{-\sqrt{x+4}}^{0} f(x,y)\,dy$$

⑦ $\int\limits_{-1}^{2} dx \int\limits_{x^2}^{2+x} f(x,y)\,dy$

$-1 \le x \le 2$

$\underbrace{x^2 \le y}_{\text{парабола}} \le \underbrace{2+x}_{\text{прямая}}$



$y = x^2$

$x = \pm\sqrt{y}$

$y = 2+x$

$x = y-2$

x	0	-2
y	2	0